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A SIMPLE METHOD FOR GRAPHICALLY OBTAINING THE COMPLEX ROOTS OF A CUBIC EQUATION

BY RUTHERFORD E. GLEASON

LET $a \pm b\sqrt{-1}$ and c denote the roots of the equation

$$x^3 + px^2 + qx + r = 0.$$

Plot the curve $x^3 + px^2 + qx + r = y$. The curve cuts the x-axis at $P \equiv (c, 0)$. If a straight edge be revolved about P as a pivot until it touches the curve as a tangent at T, the abscissa of T will be a, and the slope of the tangent will be b^2. Thus the real part, a, of the complex roots may be found immediately, and the coefficient of the imaginary unit from the fact that $b^2(a - c)$ and the ordinate of T are equal.

This is easily shown as follows:

$$(1) \qquad x^3 + px^2 + qx + r = y.$$

$$\frac{dy}{dx} = 3x^2 + 2px + q.$$

The tangent at the point (x_1, y_1) is therefore

$$(2) \qquad y - y_1 = (3x_1^2 + 2px_1 + q)(x - x_1).$$

Since $a \pm b\sqrt{-1}$ and c are the roots of $x^3 + px^2 + qx + r = 0$,

$$p = -2a - c, \qquad q = a^2 + 2ac + b^2, \qquad r = -a^2c - b^2c.$$

Putting $x = a$ and substituting the values of p, q, r in (1),

$$y = b^2(a - c).$$

Putting $x_1 = a$, $y_1 = b^2(a - c)$, and substituting the values of p and q in (2), we have

$$y = b^2(x - c)$$

as the tangent at $T \equiv (a, b^2(a - c))$, from which it is seen that the tangent at T passes through P.

When the curve has a maximum and a minimum, if the minimum is above the X-axis, a is greater than c and the ordinate of T is positive. Hence $b^2(a-c)$ is positive, and b is therefore real; but if the finite maximum is below the X-axis, a is less than c and the ordinate of T is negative. Hence $b^2(a-c)$ is negative and, since $a-c$ is negative, b is real. In the third case the X-axis is cut in three places and we may choose any of the three intersections for c. If the intersection corresponding to the root whose value is algebraically least is chosen, and a is greater than c, then $b^2(a-c)$ is negative; but since $a-c$ is positive, b is a pure imaginary. If the middle intersection is taken for c, $b^2(a-c)$ is either positive or negative and a is greater or less than c, in such wise that b is always a pure imaginary. If the middle intersection, when it is taken for c, is at a point of inflection, $b^2(a-c)=0$ and $a=c$. In this case b cannot be determined graphically. If the third intersection is taken for c, $b^2(a-c)$ is positive and a is less than c, and b is therefore a pure imaginary. Hence in the third case $a \pm b\sqrt{-1}$ is real. It is seen that the third case furnishes an interesting check in graphically obtaining the roots of the cubic when they are all real. If the curve possesses no maximum and minimum we may substitute *point of inflection* for both maximum and minimum in the foregoing discussion.

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